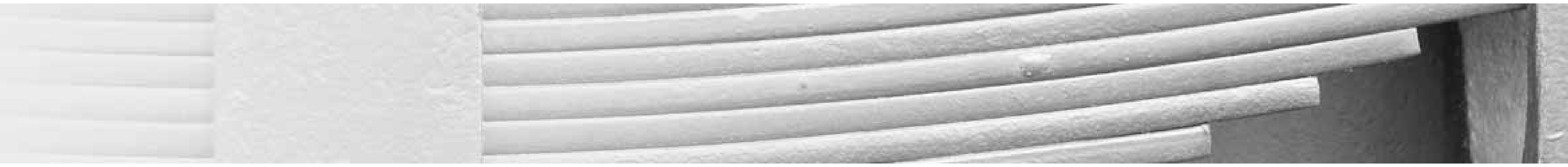


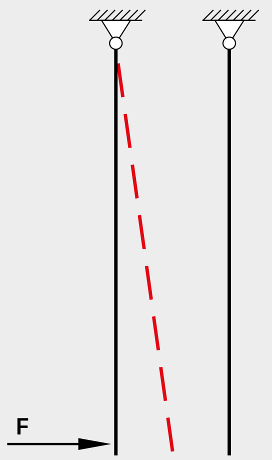
Basic knowledge

Stability problem buckling

If slim and long components such as bars, beams and columns are subject to compressive stress owing to a force along the bar axis, these can end up in indifferent or unstable equilibrium positions. If the force **F** is less than the critical force $\mathbf{F_K}$, also known as buckling force, the component is in a stable equilibrium position and there is a strength problem. If the force **F** reaches the buckling force $\mathbf{F_K}$ of the bar, the bar suddenly starts to buckle. The components, thus, lose their ability to function. Buckling is usually a very sudden and abrupt process which causes large deformations.

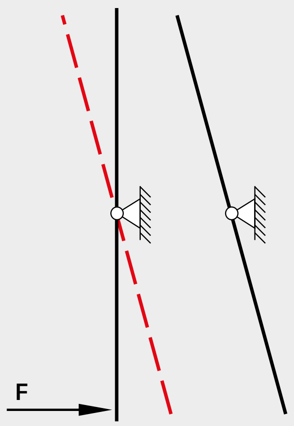
Different equilibrium positions

Stable equilibrium



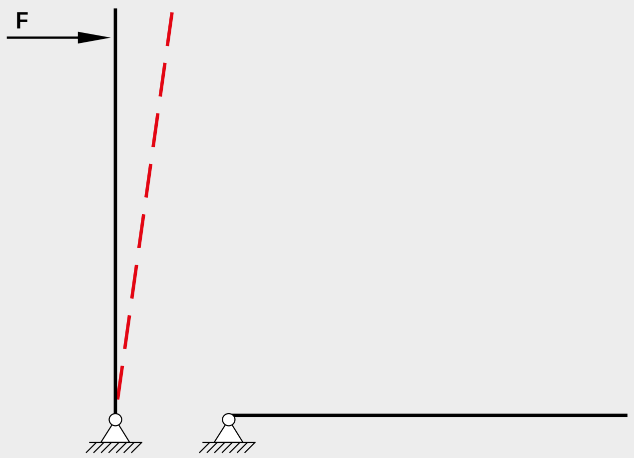
Bar returns to its starting position after the load is removed.

Indifferent equilibrium



Bar remains in the new position after the load is removed.

Unstable equilibrium



Bar does not return to the starting position after the load is removed and does not stay in the position it assumed while the load is being applied. The bar falls over.

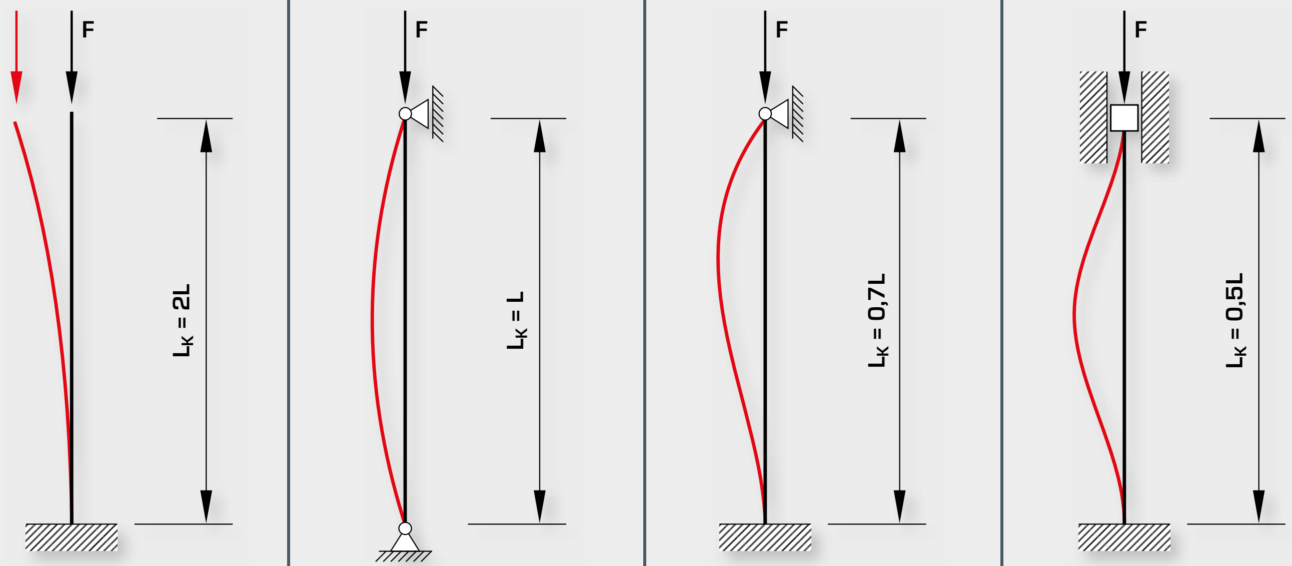
F force

Stability in bars

Bars under pressure are a typical stability problem. Here, we investigate when a straight bar collapses. The critical buckling force $\mathbf{F_K}$ describes the smallest possible compressive force under which the bar buckles. The critical buckling stress $\boldsymbol{\sigma_K}$ is the stress that occurs at the critical buckling force $\mathbf{F_K}$. The buckling force for pressure-loaded bars depends on the support conditions, bending stiffness and geometry of the shape of the bar cross-section. Euler's four buckling cases are taken as the basis for the study of the bending stability of bars with constant bending stiffness.

Euler's buckling cases

The mathematician and physicist Leonhard Euler defined four typical buckling cases to calculate the buckling force. For each of these cases, there is a buckling length coefficient **β** that is used to determine the buckling length $\mathbf{L_K}$.



Case 1:
one bar end fixed,
one bar end free

buckling length
coefficient **β = 2**
buckling length $\mathbf{L_K = L \cdot \beta}$

Case 2:
both bar ends pinned

buckling length
coefficient **β = 1**
buckling length $\mathbf{L_K = L \cdot \beta}$

Case 3:
one bar end fixed,
one bar end pinned

buckling length
coefficient **β ≈ 0,7**
buckling length $\mathbf{L_K = L \cdot \beta}$

Case 4:
both bar ends fixed

buckling length
coefficient **β ≈ 0,5**
buckling length $\mathbf{L_K = L \cdot \beta}$

F force, **L** bar length, $\mathbf{L_K}$ buckling length, **β** buckling length coefficient

Determining the buckling force F_K

$$F_K = \frac{\pi^2 \cdot E \cdot I}{L_K^2}$$

$\mathbf{F_K}$ critical buckling force, $\mathbf{L_K}$ bar length, **E** elastic modulus, **I** axial second moment of the cross-section area

Determining the buckling stress σ_K

To determine the buckling stress we use the degree of slenderness **λ** as a material parameter and the moment of area radius **i**.

$$\lambda = \frac{\beta \cdot L}{i} \qquad \sigma_K = \frac{\pi^2 \cdot E}{\lambda^2}$$

$$i = \sqrt{\frac{I}{A}}$$

$\boldsymbol{\sigma_K}$ buckling stress, **E** elastic modulus, **λ** degree of slenderness, **β** buckling length coefficient, **L** bar length, **i** moment of area radius, **A** cross-section area of the buckled bar, **I** second moment of area